Vectors 6.3 - 6.4

def: a directed line segment with an initial point, a terminal point, magnitude (aka length)

Component Form $P = (p_1, p_2)$ $Q = (q_1, q_2)$

$$\overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle = v$$

$\langle 3, 5 \rangle$

magnitude

$$\|v\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{v_1^2 + v_2^2}$$

if $\|v\| = 1$, then v is a unit vector

Vector Operations

$$u + v = \langle u_1 + v_1, u_2 + v_2 \rangle$$

$$ku = \langle ku_1, ku_2 \rangle$$

k is a constant

ex $v = \langle -2, 5 \rangle$ $w = \langle 3, 4 \rangle$

a) $2v$

$\langle -4, 10 \rangle$

b) $w - v$

$\langle 5, -1 \rangle$

c) $v + 2w$

$\langle 4, 13 \rangle$

d) $2v - 3w$

$\langle -4, 10 \rangle - \langle 9, 12 \rangle$

$\langle -13, -2 \rangle$

Properties of Vectors

1. $v + u = u + v$
2. $(u + v) + w = u + (v + w)$
3. $u + 0 = u$
4. $u + (-u) = 0$
5. $c(du) = (cd)u$ (c + d are scalars)
6. $(c + d)u = cu + du$
7. $c(u + v) = cu + cv$
8. $1(u) = u$ and $0(u) = 0$
9. $\|cv\| = |c| \cdot \|v\|$

How to find a unit vector

u = unit vector = $\dfrac{v}{\|v\|}$

ex Find a unit vector in the direction of $v = \langle -2, 5 \rangle$

$$\|v\| = \sqrt{(-2)^2 + (5)^2} = \sqrt{4+25} = \sqrt{29}$$

$$u = \left\langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle = \left\langle \frac{-2\sqrt{29}}{29}, \frac{5\sqrt{29}}{29} \right\rangle$$

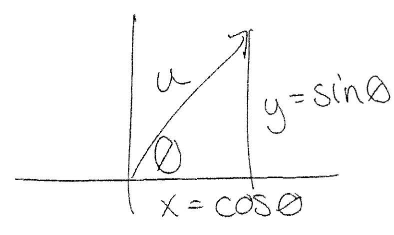
Linear Combinations of Unit Vectors

$$u = \langle -3, 8 \rangle \rightarrow -3i + 8j$$

Direction angles

$$u = \langle x, y \rangle = \langle \cos\theta, \sin\theta \rangle =$$

$$\|u\| \cos\theta\, i + \|u\| \sin\theta\, j$$



6.4 Dot Products

$u = \langle u_1, u_2 \rangle$ $\quad$ $v = \langle v_1, v_2 \rangle$

$$u \cdot v = u_1 v_1 + u_2 v_2 = \text{a scaler}$$

$u \cdot v = u_1 v_1 + u_2 v_2 =$ a scaler
a #

Properties

1. $u \cdot v = v \cdot u$
2. $0 \cdot v = 0$
3. $u \cdot (v + w) = u \cdot v + u \cdot w$
4. $v \cdot v = \|v\|^2$
5. $\boxed{c(u \cdot v)} = cu \cdot v = u \cdot cv$

* Note → don't double it!

ex Find the dot product

① $\langle 4, 5 \rangle \cdot \langle 2, 3 \rangle$

$4 \cdot 2 + 5 \cdot 3$

$8 + 15 = 23$

② $\langle 2, -1 \rangle \cdot \langle 1, 2 \rangle$

0

Given $u = \langle -1, 3 \rangle$ $v = \langle 2, -4 \rangle$

and $w = \langle 1, -2 \rangle$

1. $(u \cdot v)w$ $\qquad -1 \cdot 2 + 3 \cdot -4$
   $-14 \langle 1, -2 \rangle$ $\qquad -2 - 12$
   $\boxed{\langle -14, 28 \rangle}$ $\qquad -14$

2. $u \cdot 2v$

Angle between 2 vectors

given u & v as vectors

$$\cos\theta = \frac{u \cdot v}{\|u\| \|v\|}$$

if $\perp$ (aka orthogonal)

then $u \cdot v = 0$

ex Orthogonal? $u = \langle 2, -3 \rangle$ $v = \langle 6, 4 \rangle$